Unified Time- and Frequency-Domain Approach for Accurate Modelling of Electromagnetic Radiation Processes in Ultra-Wideband Antennas

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Abstract— **In this paper, a singularity-expansion-method-based approach is proposed for the accurate time- and frequency-domain modelling of electromagnetic field radiation processes in ultra-wideband antennas. By the use of a dedicated two-step vector fitting procedure, a minimal pole/residue spherical harmonic expansion of time-domain equivalent electric and magnetic currents excited on a suitable spherical Huygens surface enclosing the considered radiating structure has been derived. The transient electromagnetic field distribution in Fraunhofer region is presented in closed form as the superposition of outgoing propagating non-uniform spherical waves attenuating along with the radial distance and time according to the real part of the poles accounting for complex resonant processes occurring in the structure. The proposed approach has been validated by application to an ultra-wideband resistively-loaded bow-tie antenna for ground-penetrating radar applications.**

I. Introduction

Characterization of ultra-wideband (*UWB*) antenna radiation processes in far-field region is a time consuming procedure, which requires performing the numerical computation of radiation integrals involving free-space dyadic Green's functions. This is usually carried out in the frequency domain, and the spatial distribution of the electromagnetic field is typically presented by a set of radiation patterns at different frequencies. However, such an approach does not provide an integral physical insight into the mechanisms which are responsible for the electromagnetic behaviour of the structure, and requires computation and storage of a large amount of data (think about a series of three-dimensional radiation patterns over a large frequency band). Some attempts have been made in the recent past to come to more compact semi-analytical representations of the electromagnetic field distribution excited by complex *UWB* antennas [1].

In this paper a completely different approach, generalizing and expanding the model in [2], is proposed. The radiated field is presented directly in the time domain as the superposition of outgoing propagating non-uniform spherical waves related to the complex resonant processes occurring in the structure. To this end, any time-domain integral-equation, or finite-difference technique can be adopted to carry out the full-wave analysis within a volume surrounding the antenna, and to determine on-the-fly in step with the numerical simulation a spherical harmonic expansion of the equivalent electric and magnetic currents excited on a suitable Huygens surface enclosing the radiating structure. Then, a dedicated time-domain vector fitting procedure is used to derive a pole/residue representation of the aforementioned currents, and to conveniently evaluate the electromagnetic field distribution in Fraunhofer region without performing the numerical computation of radiation integrals involving free-space dyadic Green's functions. In this way, one can easily determine antenna far-field parameters of interest, such as gain and effective height, in both the time and frequency domains.

The paper is organised as follows. Section II describes the singularity-expansion method (*SEM*) for modelling of transient radiation phenomena. Section III introduces a dedicated time-domain vector fitting procedure for field-related quantities. The application of the proposed technique to the analysis of an ultra-wideband resistively-loaded bow-tie antenna for ground-penetrating radar (*GPR*) applications is then presented in Section IV. Conclusions are finally summarised in Section V.

II. Pole/Residue Modelling of Transient Electromagnetic Radiation Processes

Let us consider an antenna operating in free space, and enclosed by a spherical surface S_h having radius R_h. Naturally induced by S_h in three-dimensional space is the polar coordinate system (r,ϑ,φ) useful to express the physical quantities of interest. Denoting by $\mathbf{E}(\mathbf{r},t)$, $\mathbf{H}(\mathbf{r},t)$ the electric and magnetic field, respectively, excited by the considered radiating structure at any observation point $\mathbf{r} \equiv \{r,\vartheta,\varphi\}$, and any time $t \geq 0$, the Huygens currents on S_h can be easily evaluated as:

$$\begin{cases} \mathbf{J}_S(\vartheta,\varphi,t) = \hat{\boldsymbol{r}} \times \mathbf{H}(R_h,\vartheta,\varphi,t)\,, \\ \mathbf{M}_S(\vartheta,\varphi,t) = \mathbf{E}(R_h,\vartheta,\varphi,t) \times \hat{\boldsymbol{r}}\,. \end{cases} \tag{1}$$

Hence, by virtue of the surface equivalence theorem, the spatial distribution of the electromagnetic field in Fraunhofer region can be determined by convolving (1) with the radiative component of free-space dyadic Green's functions expressed in spherical coordinates. In this way, it follows, after some algebra:

$$\mathbf{E}(\mathbf{r},t) \simeq \frac{1}{4\pi r c_0}\hat{\boldsymbol{r}}\times\iint_{S_h}\left[\eta_0\hat{\boldsymbol{r}}\times\dot{\mathbf{J}}_S\left(\vartheta',\varphi',\tau+\frac{R_h}{c_0}\cos\gamma\right)+ + \dot{\mathbf{M}}_S\left(\vartheta',\varphi',\tau+\frac{R_h}{c_0}\cos\gamma\right)\right]dS' , \tag{2}$$

$$\mathbf{H}(\mathbf{r},t) \simeq \hat{\boldsymbol{r}}\times\mathbf{E}(\mathbf{r},t)\,/\,\eta_0 , \tag{3}$$

where $\cos\gamma = \sin\vartheta\sin\vartheta'\cos(\varphi-\varphi')+\cos\vartheta\cos\vartheta'$, and $\tau = t - r/c_0$ is the normalized time.

The radiation integral appearing in (2) can be conveniently handled by applying a two-step data-fitting procedure aimed to the expansion of equivalent currents $\mathbf{J}_S(\vartheta,\varphi,t)$ and $\mathbf{M}_S(\vartheta,\varphi,t)$ in terms of time-variant vector spherical harmonic functions with proper coefficients. At each time t , the aforementioned currents are expressed as the linear combination of orthonormalized harmonics, defined as:

$$Y_n^m(\vartheta,\varphi) = \sqrt{\frac{2n+1}{4\pi}\frac{(n-m)!}{(n+m)!}}\,P_n^m(\cos\vartheta)\,e^{-jm\varphi} , \tag{4}$$

$P_n^m(\cdot)$ being the associated Legendre function of the first kind of degree n and order m . In particular, it can be shown that $Y_n^{-m}(\vartheta,\varphi) = (-1)^m Y_n^{m^*}(\vartheta,\varphi)$, where the superscript * denotes complex conjugation. Hence, one can readily obtain:

$$\begin{Bmatrix}\mathbf{J}_S(\vartheta,\varphi,t)\\ \mathbf{M}_S(\vartheta,\varphi,t)\end{Bmatrix} = \sum_{n=0}^{+\infty}\sum_{m=-n}^{n}\begin{Bmatrix}\boldsymbol{\alpha}_{n,m}(t)\\ \boldsymbol{\beta}_{n,m}(t)\end{Bmatrix}Y_n^m(\vartheta,\varphi) , \tag{5}$$

where the vector expansion coefficients are computed as:

$$\begin{Bmatrix}\boldsymbol{\alpha}_{n,m}(t)\\ \boldsymbol{\beta}_{n,m}(t)\end{Bmatrix} = \int_0^{2\pi}\int_0^{\pi}\begin{Bmatrix}\mathbf{J}_S(\vartheta,\varphi,t)\\ \mathbf{M}_S(\vartheta,\varphi,t)\end{Bmatrix}Y_n^{m^*}(\vartheta,\varphi)\sin\vartheta\, d\vartheta\, d\varphi . \tag{6}$$

Series representation (5) is exact as long as n goes to infinity. Truncation errors will arise when limiting the sum over n to a finite discrete angular bandwidth N . For sake of brevity, the optimal selection of N , as well as the developed semi-analytical procedure to compute integrals in (6) is not addressed here, and will be discussed in future papers.

Provided that the considered antenna is excited by a finite-duration signal, the current expansion coefficients in (6) can be represented using the following *SEM*-based expression:

$$\begin{Bmatrix}\boldsymbol{\alpha}_{n,m}(t)\\ \boldsymbol{\beta}_{n,m}(t)\end{Bmatrix} = \sum_{k=1}^{K}\begin{Bmatrix}\mathbf{a}_{n,m,k}\\ \mathbf{b}_{n,m,k}\end{Bmatrix}e^{s_{n,m,k}t}u(t) + \begin{Bmatrix}\mathbf{A}_{n,m}(t)\\ \mathbf{B}_{n,m}(t)\end{Bmatrix} , \tag{7}$$

where $s_{n,m,k} = -\sigma_{n,m,k} + j\omega_{n,m,k}$ and $\mathbf{a}_{n,m,k}$, $\mathbf{b}_{n,m,k}$ are the complex poles and vector residues, respectively, of the exponential terms accounting for damped natural resonant processes occurring in the structure. From the *SEM* theory it is known that the entire functions $\mathbf{A}_{n,m}(t)$, $\mathbf{B}_{n,m}(t)$ are needed to accurately describe early-time behaviour of the antenna. However, as discussed in [2], such terms are usually neglected, and their contribution efficiently taken into account by means of a modified pole/residue representation.

Finally, substituting equations (5) and (7) into equations (2)-(3) yields:

$$\begin{Bmatrix}\mathbf{E}(r,\vartheta,\varphi,t)\\ \mathbf{H}(r,\vartheta,\varphi,t)\end{Bmatrix} \simeq \frac{R_h^2}{4\pi r c_0}\sum_{n=0}^{N}\sum_{m=-n}^{n}\sum_{k=1}^{K}s_{n,m,k}e^{s_{n,m,k}\tau}\begin{Bmatrix}\mathbf{e}_{n,m,k}\\ \mathbf{h}_{n,m,k}\end{Bmatrix}\cdot \cdot\iint_{\Omega_h}e^{\frac{s_{n,m,k}}{c_0}R_h\cos\gamma}Y_n^m(\vartheta',\varphi')\sin\vartheta' d\vartheta' d\varphi' , \tag{8}$$

where

$$\begin{Bmatrix}\mathbf{e}_{n,m,k}\\ \mathbf{h}_{n,m,k}\end{Bmatrix} = \hat{\boldsymbol{r}}\times\begin{Bmatrix}\eta_0\hat{\boldsymbol{r}}\times\mathbf{a}_{n,m,k} + \mathbf{b}_{n,m,k}\\ -\mathbf{a}_{n,m,k} + \hat{\boldsymbol{r}}\times\mathbf{b}_{n,m,k}\,/\,\eta_0\end{Bmatrix} . \tag{9}$$

In (8) $\Omega_h = \Omega_h(\vartheta',\varphi',\vartheta,\varphi,\tau)$ denotes the angular domain of the equivalent currents on S_h contributing, at the normalized time τ , to the radiated electromagnetic field value excited at the observation point $\{r,\vartheta,\varphi\}$, namely:

$$\Omega_h = \left\{(\vartheta',\varphi')\in[0,\pi]\times[0,2\pi] : \cos\gamma > -c_0\tau/R_h\right\} . \tag{10}$$

As it can be easily recognized, $\Omega_h = [0,\pi]\times[0,2\pi)$ for $t > T_h(r) = (r+R_h)/c_0$. So, under such assumption, by exploiting properties of harmonic and Bessel functions, radiation integrals in (8) can be evaluated in closed form as:

$$\iint_{\Omega_h}e^{\frac{s_{n,m,k}}{c_0}R_h\cos\gamma}Y_n^m(\vartheta',\varphi')\sin\vartheta' d\vartheta' d\varphi' = = 4\pi i_n\left(\frac{s_{n,m,k}}{c_0}R_h\right)Y_n^m(\vartheta,\varphi) , \tag{11}$$

$i_n(\xi) = \sqrt{\frac{\pi}{2\xi}}I_{n+1/2}(\xi)$ being the spherical modified Bessel function of order n . Consequently, the time-domain electromagnetic field radiated by the antenna in Fraunhofer region is found as the following superposition of outgoing propagating non-uniform spherical waves attenuating along with the radial distance and time according to the real part $\sigma_{n,m,k}$ of the complex poles:

$$\begin{Bmatrix}\mathbf{E}(r,\vartheta,\varphi,t)\\ \mathbf{H}(r,\vartheta,\varphi,t)\end{Bmatrix} \simeq \frac{R_h^2}{r c_0}\sum_{n=0}^{N}\sum_{m=-n}^{n}\sum_{k=1}^{K}s_{n,m,k}e^{s_{n,m,k}\tau}i_n\left(\frac{s_{n,m,k}}{c_0}R_h\right)\cdot \cdot Y_n^m(\vartheta,\varphi)u(\tau)\begin{Bmatrix}\mathbf{e}_{n,m,k}\\ \mathbf{h}_{n,m,k}\end{Bmatrix} . \tag{12}$$

Using (12) one can conveniently determine antenna far-field parameters of interest, such as gain, effective height, and transient response to any arbitrary excitation.

III. Time-Domain Vector Fitting Procedure

Let us now focus the attention on the modified pole/residue representation of the general time-variant spherical harmonic expansion coefficient $\boldsymbol{\psi}_{n,m}(t) = [\boldsymbol{\alpha}_{n,m}(t)\ \boldsymbol{\beta}_{n,m}(t)]$ relevant to the surface equivalent currents excited on S_h :

$$\boldsymbol{\psi}_{n,m}(t) = \sum_{k=1}^{K} \mathbf{c}_{n,m,k} e^{s_{n,m,k} t} u(t) , \tag{13}$$

where $\mathbf{c}_{n,m,k} = [\mathbf{a}_{n,m,k} \;\; \mathbf{b}_{n,m,k}]$. Due to the vectorial nature of the residues $\mathbf{c}_{n,m,k}$, the matrix pencil method, or modified Prony's algorithms can not be applied to perform the fitting of $\boldsymbol{\psi}_{n,m}(t)$ by complex exponential functions as in [2]. So, to overcome this limitation, a dedicated non-conventional time-domain vector fitting (*TD-VF*) procedure has been specifically developed. Such procedure, originally introduced in [3] to extract frequency-independent equivalent circuits of multiport interconnect structures, is aimed to the derivation of a rational approximation to the Laplace transform of $\boldsymbol{\psi}_{n,m}(t)$, that is:

$$\boldsymbol{\Psi}_{n,m}(p) = \mathcal{L}\left\{\boldsymbol{\psi}_{n,m}(t)\right\} = \sum_{k=1}^{K} \frac{\mathbf{c}_{n,m,k}}{p - s_{n,m,k}} . \tag{14}$$

To this end, a scalar weight function:

$$\Sigma_{n,m}(p) = 1 + \sum_{k=1}^{K} \frac{R_{n,m,k}}{p - q_{n,m,k}} = \prod_{k=1}^{K} \frac{p - \zeta_{n,m,k}}{p - q_{n,m,k}} , \tag{15}$$

with assigned initial poles $\{q_{n,m,k}\}$, and unknown residues $\{R_{n,m,k}\}$ is introduced. In addition, let us assume that the following approximation holds:

$$\Sigma_{n,m}(p)\boldsymbol{\Psi}_{n,m}(p) \simeq \sum_{k=1}^{K} \frac{\mathbf{M}_{n,m,k}}{p - q_{n,m,k}} . \tag{16}$$

Since the right-hand-side term of (16) features the same complex poles $\{q_{n,m,k}\}$ as the weight function, a cancellation between the zeros $\{\zeta_{n,m,k}\}$ of $\Sigma_{n,m}(p)$ and the poles $\{s_{n,m,k}\}$ of $\boldsymbol{\Psi}_{n,m}(p)$ must occur. This condition provides indeed a way to estimate $\{s_{n,m,k}\}$ by solving in a least squares sense, for the unknown residues $\{R_{n,m,k}\}$, the following time-domain equation:

$$\begin{aligned} \sum_{k=1}^{K} e^{q_{n,m,k} t} \mathbf{M}_{n,m,k} &= \mathcal{L}^{-1}\left\{\sum_{k=1}^{K} \frac{\mathbf{M}_{n,m,k}}{p - q_{n,m,k}}\right\} \simeq \\ \simeq \mathcal{L}^{-1}\left\{\Sigma_{n,m}(p)\boldsymbol{\Psi}_{n,m}(p)\right\} &= \boldsymbol{\psi}_{n,m}(t) + \sum_{k=1}^{K} R_{n,m,k} \mathbf{I}_{n,m,k}(t) , \end{aligned} \tag{17}$$

where:

$$\mathbf{I}_{n,m,k}(t) = \int_{0}^{t} e^{q_{n,m,k}(t-t')} \boldsymbol{\psi}_{n,m}(t') dt' \tag{18}$$

is the transient waveform resulting from the inverse Laplace transform of the $k-$th partial fraction term appearing on the left-hand side of expansion (16). Once the evaluation of residues $\{R_{n,m,k}\}$ is carried out, the poles can be determined by enforcing $s_{n,m,k} = \zeta_{n,m,k}$, where the zeros $\{\zeta_{n,m,k}\}$ are computed using a suitable algorithm such as the Collins - Krandick technique. The described method, known as *pole relocation*, avoids the use of ill-conditioned non-linear least squares methods for the direct fitting of (13)-(14). In particular, the pole relocation can be iterated using the estimated poles $\{s_{n,m,k}\}$ as starting poles $\{q_{n,m,k}\}$ of the scalar weight function $\Sigma_{n,m}(p)$ at the next iteration. The convergence of the described procedure is usually obtained in a few iterations (less than ten). Finally, once the poles $\{s_{n,m,k}\}$ are known, vector residues $\{\mathbf{c}_{n,m,k}\}$ can be conveniently determined by solving (13) in a least squares sense.

IV. An Example of UWB Antenna Modelling

The considered approach has been validated by application to an *UWB* resistively-loaded bow-tie antenna for *GPR* applications. A sketch of the structure geometry is shown in Fig. 1. As it appears, the antenna features two circularly ended flairs, with half angle $\Theta_f / 2 = 37.5°$, whose electrical conductivity σ_f is function of the radial distance ρ from the central delta gap. Here, a voltage source of amplitude $V_g = 1\,V$, and internal resistance $R_g = 200\,\Omega$ is employed to excite the radiating element.

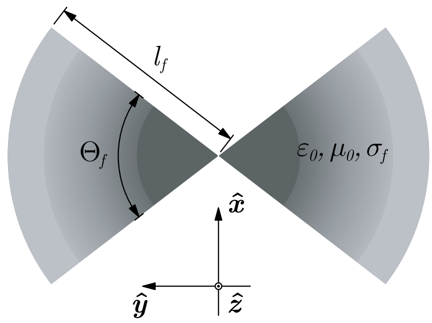

Fig. 1 Top view of a resistively loaded bow-tie antenna for *GPR* applications. Antenna characteristics: $l_f = 40\,cm$, $\Theta_f = 75°$. The Cartesian coordinate system adopted to express the field quantities is sketched.

To properly enlarge the antenna bandwidth (thus reducing late-time ringing due to spurious multiple reflections between the flairs open ends and the feed point), the relevant resistive loading has been assumed to have the following Wu-King-like distribution:

$$\sigma_f(\rho) = \sigma_0 \frac{1 + \sigma_{\min}/\sigma_0 - \rho / l_f}{\rho / l_f} , \tag{19}$$

where $l_f = 40\,cm$ is the flairs length, and $\sigma_{\min} = 1\,mS/m$ is the electrical conductivity value at the antenna end sections. In particular, the optimal parameter σ_0 appearing in (19) has been found to be equal to about $40\,S/m$.

The near-field full-wave analysis of the considered radiating structure has been performed by means of a locally conformal finite-difference time-domain (*FDTD*) scheme useful to accurately model dielectric, metal, and radiation losses avoiding the staircase approximation of the geometry [4]. The antenna has been meshed on a *UPML*-backed uniform cubic grid with spatial increment

$\Delta h = \lambda_{-10dB}/30 \simeq 5\ mm$, where λ_{-10dB} is the free-space wavelength at the upper $-10dB$ cut-off frequency, $f_{-10dB} = 2\ GHz$, of the excitation voltage signal, which is the Gaussian pulse defined by:

$$\Pi_g(t) = V_g\, e^{-(t-t_0)^2/\tau_g^2}\ , \tag{20}$$

where $t_0 = 10\tau_g$, and

$$\tau_g = \sqrt{\ln 10}\Big/\left(\pi f_{-10dB}\right) \simeq 0.24\ ns\ . \tag{21}$$

The source pulse is coupled into the finite-difference equations used to update the time-domain electric field distribution within the delta gap.

It has been numerically found that the antenna with the specified resistive loading is well-matched to the feeding line in the frequency band from $f_{low} \simeq 130\ MHz$ to $f_{high} \simeq 2.9\ GHz$. This excellent performance is useful to meet demanding specifications of modern *GPR* applications both in terms of fractional bandwidth, and low-frequency operation.

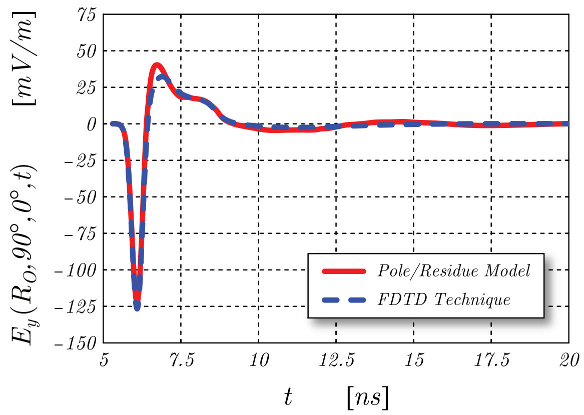

Fig. 2 Transient behaviour of the co-polarized electric field component excited by the resistively loaded bow-tie antenna at the observation point $\mathbf{r} \equiv \{R_O, 90°, 0°\}$ ($R_O = 1.1\ m$), as computed by a locally conformal *FDTD* technique and the proposed pole/residue model with expansion orders $N = 10$ and $K = 20$.

Using the developed *SEM*-based approach, the time-variant spherical harmonic expansion vector coefficients $\{\psi_{n,m}(t)\}$ of the surface equivalent currents excited on the Huygens sphere S_h with radius $R_h = 45\ cm$ have been computed on-the-fly in step with the numerical *FDTD* simulation, and then fitted to the modified pole/residue expansion (13) with order $K = 20$. According to the diameter of S_h, the Fraunhofer distance, for $T_g = 10\tau_g \simeq 2.4\ ns$, has been found to be $r_F \simeq R_O = 1.1\ m$. Hence, the proposed non-uniform spherical wave representation (12) with order $N = 10$ has been applied to evaluate the transient radiated electromagnetic field distribution excited at distance $r = R_O$ for times $t > T_h(R_O) = (R_O + R_h)/c_0 \simeq 5.2\ ns$. As it can be noticed in Fig. 2, the agreement with numerical results obtained using the full-wave locally conformal *FDTD* technique is very good.

V. Conclusions

A *SEM*-based approach has been developed for the accurate time- and frequency-domain characterization of radiation processes in ultra-wideband antennas. Any time-domain integral-equation, or finite-difference technique can be adopted to compute the spatial distribution of the electromagnetic field within a volume surrounding the antenna under analysis, and to determine on-the-fly in step with the numerical simulation a spherical harmonic expansion of the equivalent electric and magnetic currents excited on a suitable Huygens surface enclosing the radiating structure. By the use of a dedicated two-step vector fitting procedure, a minimal pole/residue spherical harmonic expansion of the aforementioned time-domain equivalent currents has been derived. The proposed approach allows evaluating in closed form the transient electromagnetic field distribution in Fraunhofer region as the superposition of outgoing propagating non-uniform spherical waves attenuating along with the radial distance and time according to the real part of the poles accounting for complex resonant processes occurring in the structure. Hence, antenna far-field parameters of interest, such as directivity and effective height, can be easily determined in both the time and frequency domains, without performing the numerical computation of radiation integrals involving free-space dyadic Green's functions. In this way, one can gain a meaningful insight into the physical mechanisms which are responsible for the electromagnetic behaviour of the structure. Such information can be usefully exploited to optimize the performance of *UWB* antennas for a wide variety of applications.

The considered model has been validated by application to a resistively-loaded bow-tie antenna. The transient response of the structure evaluated using a minimal pole/residue model has been found to be in good agreement with the reference solution obtained by means of a full-wave locally conformal *FDTD* technique. The computational time and memory usage required to carry out the pole/residue spherical harmonic expansion are negligible in comparison with those relevant to the *FDTD* numerical simulation for computing the far-field distribution.

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